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Open string engineering of D-brane geometry

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Abstract: One-loop scattering on a stack of D3 branes was considered in arXiv:0801.0218 [hep-th]. Divergence was found and its cancelation mechanism was proposed, wherein it was conjectured that the D-brane geometry be introduced in the form of counter vertex operators. Here we verify the conjecture at the first few leading orders in an expansion method that we call large-r_0 expansion. We comment on the relation with the Fischler-Susskind mechanism and discuss the implications of our result for AdS/CFT.

Keywords: Gauge-gravity correspondence, D-branes, AdS-CFT Correspondence.

JHEP08(2008)026

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Contents

JHEP08(2008)026

1. Introduction

A D-brane is a hyperplane where the end points of an open string can be attached [1]. When two or more open strings come across they will scatter each other. Studying the scattering will be interesting and relevant for several reasons.[1] For example, knowledge on scattering may shed some light on the better understanding of AdS/CFT and its derivation, which has been our main motivation. The derivation in turn may provide a new paradigm for the unification of gauge theory and gravity. When a phenomenologically more realistic model of a D-brane configuration becomes available it may also be necessary to consider scattering of states not only at the low energy field theory level (which may or may not be renormalizable) but also at the full level of open string.

For the actual study one must first construct the vertex operators for the external scattering states. This has been carried out in the GS formulation in one of our previous works taking the D3 brane case as an example [3]. On a D3 brane the D9-brane multiplet gets resolved into two multiplets which we call the scalar multiplet and the vector multiplet in analogy with the N=2 susy field theory. Subsequently various tree and one-loop

[1] In the past scattering on D-branes was studied in the NSR formulation by several authors [2]. We will use the Green-Schwarz (GS) formulation which is almost inevitable for the things that we try to do.

amplitudes were computed [4]. One loop divergence structure was obtained. It was noted that the divergence structure does not share the nice feature of the D9 brane, which seems to suggest that it may require a more radical measure to remove. The deviation from the D9 brane is due to the different structure of the zero modes. A proposal for cancelation of the divergence was put forward in [4]: it should be possible to absorb the divergence by adding "counter vertex operators" of composite nature. They are to be constructed out of the open string fields.

It was conjectured that the precise forms of the vertex operators will have a link to the geometry induced by the D-branes.[2] According to the conjecture the geometry should provide a guidance to find the counter vertex operators. It would be a hard task to construct them without such aid. Once fixing the form of the counter vertex operator the geometry should be taken as an out-come: it arises *as a result of the flat space analysis*. It is a secondary by-product of open string loop effects, hence the title of the paper. Nowhere in the construction the explicit closed string degrees of freedom are used. The composite vertex operators might be interpreted as representing a closed string state but that is, together with the by-product geometry, as close as it gets to the close string. The whole construction so far is based on the purely open string frame work.

Some preliminary computation was presented in [4] on the amplitudes with the counter vertex operators inserted. In this work we initiate a much more systematic verification of the conjecture focusing on two cases, the four scalar amplitude and the four vector amplitude. The amplitudes at tree and one loop have been obtained previously without the insertion of the couner vertex operators: here we compute at tree level

$$\langle V_s V_s V_s V_s\, V_G\rangle \quad \text{and} \quad \langle V_g V_g V_g V_g\, V_G\rangle \tag{1.1}$$

where V_s (V_g) denotes the scalar vertex operator (the vector vertex operator) and V_G the counter vertex operator. The subscript "G" represents its proposed origin, the geometry. We interchangeably call the counter vertex operator the geometry vertex operator.

The detailed construction of V_G is presented in the appendix. The basic idea is to start from the GS action in a generic curved background. The action was constructed relatively recently in [8 − 10]. In place of each supergravity field one substitutes the supergravity solution for the D3 brane geometry. For a perturbative analysis one makes an expansion which we call large r_0-expansion wherein one introduces coordinates, $X^m = X_0^m$, $r_0^2 = \sum_m X_0^m X_0^m$, for a location that is far away from the center branes. Then one makes an expansion of the resulting non-linear sigma model action around that point. Then one identifies the fields in the external scattering states as the fluctuation fields in the shifted coordinates. Why are such an expansion and identification necessary? From a practical viewpoint the large-r_0 expansion seems inevitable for the perturbative computation. Put differently the connection between the geometry and the loop effects are made where it can be made in the brane geometry. Obviously it is not the near-horizon region that we look

[2] A direct connection between the quantum effects and geometry is not entirely new. For example the map obtained in [5] can be interpreted in this context. The link between divergence and geometry goes back to the Fischler-Susskind mechanism [6, 7]. We comment on the relation in the conclusion.

JHEP08(2008)026

into. None the less one can make a connection to AdS geometry and AdS/CFT once one establishes the role of D-brane geometry. We will have more remarks on this in the conclusion.

As stated above our intention is to exclusively use the open string degrees of freedom. For one thing it would be more economical, at least from the standpoint of unifying degrees of freedom, than using the closed string degrees of freedom as well. However, as for the geometry one may question whether it really is necessary. In other words, wouldn't it be possible to cancel the divergence using a flat space action since that would be much simpler if things can work that way? The fact that the D9 brane way of canceling the divergence (i.e., by shifting the string tension [11]) does not work for the D3 brane case can be seen as follows. As in the D9 case the D3 brane one-loop produces divergence with exactly the same tree level kinematic- and gauge- structure. Let's attempt to cancel the divergence by shifting the tension of the *flat* space action. If one considers the flat space action there is no distinction between the D3 and D9 (since only the bd conditions are different): the action is

$$S = -\frac{1}{2}\int \left(T\partial_\alpha X^i \partial^\alpha X^i - \frac{i}{\pi}\bar{S}^a \rho^\alpha \partial_\alpha S^a \right) \tag{1.2}$$

The counter vertex operator that results as a consequence of varying the string tension is $\partial_\tau X^i \partial\tau X^i - \partial_\sigma X^i \partial\sigma X^i = \partial_\tau X^u \partial_\tau X^u - \partial_\sigma X^m \partial_\sigma X^m$ where we have omitted irrelevant factors. The missing terms have dropped due to the fact that the vertex operators are considered at $\sigma = 0$. Recall that the one loop results are such that the scalar four point and the vector four point amplitudes have the same signs. It implies that with the given relative signs one can not cancel the divergence of the scalar loop and the vector loop at the same time. It is to be contrasted with how things go in the D3-brane background. There the additional sign comes about, as we will see, due to the curved metric factors, $H^{1/2}$ and $H^{-1/2}$, making the geometry vertex operator $-\frac{q}{2}\partial_\tau X^\mu \partial\tau X^\mu - \frac{q}{2}\partial_\sigma X^m \partial\sigma X^m$ in the leading order. It can be read off from the quadratic order action in fermion,

$$\begin{aligned} &-\frac{1}{2}\sqrt{h}\, h^{ij}\left(\partial_i X^u \partial_j X^v \eta_{uv}(H^{-1/2}-1) + \partial_i X^m \partial_j X^n \eta_{mn}(H^{1/2}-1) \right) \\ &-\frac{i}{p^+}(\sqrt{h}\, h^{ij} - \varepsilon^{ij})\partial_i X^+ (H^{-1/4}-1)(S\partial_j S) \end{aligned} \tag{1.3}$$

With the flip of the sign the counter vertex operator has a potentially right form to work, and it does work as we will see below. Note also that the curved space provides the needed factor of the open string coupling constant, g, through q (defined in (2.3) below) automatically. The sign contradiction alone is sufficient to rule out the flat space action. But there are some other ominous features that makes the flat action unlikely. For example, since the string tension T appears only in the bosonic part the fermionic coordinates will not play a role what so ever in any arbitrary order. Although the fermionic term does not seem to play a role either in the examples that we consider in this work,[3] it simply can not be true in general. Another unfavorable feature of the flat space counter vertex operator is associated with the two loop structure. The final form of a two-loop four point amplitude will have a single integration as far as the world-sheet locations of the vertex operators are concerned as

[3] Remember that we are considering a first few leading orders of particular cases.

JHEP08(2008)026

with the corresponding one-loop amplitude. In attempt to cancel the divergence one would have to insert two vertex operators of the form $\sim g^4\partial X(y_1)\partial X(y_1)\partial X(y_2)\partial X(y_2)$. Therefore even after performing one of the two y-integration there is one too many integration compared with the two-loop result: it seems unlikely that the flat space operator will succeed. On the contrary the geometry after the large-r_0 expansion naturally produces a term at $q^2 \sim g^4$-order at a single y-location. Therefore, it seems that the flat space option is ruled out. The real question is whether the D-brane geometry does the job and, if not, what else.

The rest of the paper is organized as follows. In sec 2, we make a summary of results and present some of the salient features of the computations detailed in sec 3. We point out that the results are verification, at leading orders, of the conjecture that was put forward in [4]. In section 3, we begin by putting together several ingredients for the forthcoming computations through a brief review. We quote the full expression (i.e., expression prior to the large r_0 expansion) of the geometry vertex operator that is obtained in the appendix. After the large-r_0 expansion we carry out the four point amplitude computation for the scalar multiplet and the vector multiplet for the first two orders of the expansion. Partial results are mentioned on the third leading order. Many parts of the computations are prohibitively long for manual computation. Much of the computation has been Mathematica-coded. In the conclusion we discuss various issues such as the implications of our results for AdS/CFT (especially the stronger form thereof), some of the loose points raised in the main body, future directions. In the appendix we outline how to obtain the geometry vertex operator.

2. Summary of results

The section that follows the present one contains lengthy and tedious pieces of computations. It may be a good idea to have a summary of the results before we embark on heavy computation. To prove the conjecture, first we must show that the correlator $\langle VVVV\ V_G\rangle$ has precisely the same kinematic and momentum structure as the corresponding one loop (and tree since there are the same) result of $\langle VVVV\rangle$. The computation below seems to suggest a pattern on how this is achieved: a first few leading order terms in V_G alone produce the desired structure with the higher order terms yielding vanishing contributions.

The result of the appendix, (A.10), suggests the following form of the counter vertex operator with S being the fermionic coordinate,

$$\begin{aligned}
\pi V_G = &\ \frac{1}{2}\sqrt{h}\ h^{ij}\Big(\partial_i X^u\partial_j X^v\eta_{uv}(H^{-1/2}-1)+\partial_i X^m\partial_j X^n\eta_{mn}(H^{1/2}-1)\Big)\\
&+\frac{1}{2p^+}\Big\{-2i(\sqrt{h}\ h^{ij}-\varepsilon^{ij})\partial_i X^+(H^{-1/4}-1)(S\partial_j S)\\
&\qquad+\frac{i}{4}(\sqrt{h}h^{ij}-\varepsilon^{ij})\partial_i X^+H^{-7/4}\frac{H'}{r}\partial_j X^u X^m\ (S\gamma^{um}S)\\
&\qquad-\frac{i}{4}(\sqrt{h}h^{ij}-\varepsilon^{ij})\partial_i X^+H^{-5/4}\frac{H'}{r}\partial_j X^m X^n\ (S\gamma^{mn}S)\Big\}\\
&+\frac{1}{4(p^+)^2}\sqrt{h}h^{ij}\partial_i X^+\partial_j X^+\ H^{-1/2}\Big\{-\frac{17}{1536}\kappa_1(S\gamma^{uv}S)(S\gamma^{uv}S)
\end{aligned}$$

JHEP08(2008)026

$$
\begin{aligned}
&+\left[\frac{43}{768}\kappa_1+\frac{1}{192}\kappa_2\right](S\gamma^{au}S)(S\gamma^{au}S)\\
&-\left[\frac{1}{192}\kappa_2+\frac{1}{128}\kappa_1\right](S\gamma^{ab}S)(S\gamma^{ab}S)\\
&+X^aX^b\frac{1}{r^2}\left[\frac{31}{768}\kappa_1-\frac{1}{32}\kappa_2\right](S\gamma^{au}S)(S\gamma^{bu}S)\\
&+X^aX^b\frac{1}{r^2}\left[+\frac{1}{32}\kappa_2+\frac{29}{384}\kappa_1\right](S\gamma^{ac}S)(S\gamma^{bc}S)\Big\}
\end{aligned}
\tag{2.1}
$$

where

$$
\kappa_1=H^{-5/2}(H')^2,\qquad \kappa_2=H^{-3/2}H'\frac{1}{r},\qquad H(X^m)=1+\frac{4\pi g^2\alpha'^2}{r^4} \tag{2.2}
$$

In the right hand side of the third equation of (2.2) we have replaced the closed string coupling constant by the open string coupling constant. For a perturbative approach we expand the operator around a point, X_0^m with $(X_0^m)^2\equiv r_0^2$, that is far away from the center branes. Because of the SO(6) rotational symmetry of the brane configuration the individual coordinate X_0^m will only appear through r_0 which we will fix later. To illustrate the large r_0 expansion consider the function H. Define

$$
r_0^4=\Lambda^4\,\alpha'^2,\quad q=\frac{4\pi g^2}{\Lambda^4} \tag{2.3}
$$

where Λ is a dimensionless parameter that measures the norm of r_0 in terms of $\sqrt{\alpha'}$. Shifting $X^m\to X^m+X_0^m$ one gets

$$
H(X+X_0)=\ 1+q-\frac{4\,q\,X_0\cdot X}{r_0{}^2}+q\left(\frac{-2\,r^2}{r_0{}^2}+\frac{12\,(X_0\cdot X)^2}{r_0{}^4}\right)+\cdots \tag{2.4}
$$

It is nice to note that due to the dimensional regularization only a finite number of terms contribute for a fixed number of external states and a fixed space-time loop order. For example in the case of four point scattering we should expand up to (and including) X^4-order: higher order terms do not make contributions.[4] The expansion parameters are taken as $\frac{1}{r_0}$ (or Λ) and q. Since we are dealing with the one-loop divergence, only the linear terms in q may be kept. It seems that in the leading order of $\frac{1}{r_0}$ all of the S-quartic terms drop basically because of the fermionic equation of motion.

2.1 Scalar multiplet scattering

The kinematic structure of the one-loop divergence [11, 4] is

$$
\langle V_sV_sV_sV_s\rangle\sim\frac{1}{\epsilon}\frac{1}{4}(su\,\xi_1\cdot\xi_4\,\xi_2\cdot\xi_3+tu\,\xi_1\cdot\xi_2\,\xi_3\cdot\xi_4+st\,\xi_2\cdot\xi_4\,\xi_1\cdot\xi_3) \tag{2.5}
$$

[4]For the purely vector multiplet scattering it is even simpler since a longitudinal coordinate, X^u, and a transverse coordinate, X^m, do not contract each other: one can simply set $X^m=X_0^m$. Incidentally, this does not make the vector case simpler. The reason is that the external states come with e^{ikX}-factor which contains the longitudinal coordinates. For the q-order four point amplitudes that we consider only the quadratic terms contribute.

JHEP08(2008)026

where ϵ is a infinitesimal parameter. What we want to show, therefore, is

$$\langle V_s V_s V_s V_s \; V_G \rangle \sim \frac{1}{4}(su\;\xi_1\cdot\xi_4\;\xi_2\cdot\xi_3 + tu\;\xi_1\cdot\xi_2\;\xi_3\cdot\xi_4 + st\;\xi_2\cdot\xi_4\;\xi_1\cdot\xi_3) \tag{2.6}$$

Here and below we have suppressed a common factor $\frac{\Gamma(-\alpha' s)\Gamma(-\alpha' t)}{\Gamma(1-\alpha' s-\alpha' t)}$. It is a necessary condition. Making it sufficient will give a relation between ϵ, Λ and ϵ_y as we will discuss towards the end of sec 2. We define ϵ_y below. We break V_G into the power series expansion in $\frac{1}{r_0}$,

$$V_G = V_{G,r_0^{-4}} + V_{G,r_0^{-5}} + V_{G,r_0^{-6}} + \cdots \tag{2.7}$$

As indicated the leading order vertex operator comes with $\frac{1}{r_0^4}$. We will work out the explicit form below and show that

$$\pi V_{G,r_0^{-4}} \;= \frac{q}{4}\;\left(-\partial_\sigma X^m \partial_\sigma X^m - \partial_\tau X^u \partial_\tau X^u + il^2\left(-S\partial_\tau S - S\partial_\sigma S\right)\right) \tag{2.8}$$

With this one gets

$$\langle V_s V_s V_s V_s \; V_{G,r_0^{-4}} \rangle = \frac{4\pi g^2}{\epsilon_y \Lambda^4}\frac{1}{4}(su\;\xi_1\cdot\xi_4\;\xi_2\cdot\xi_3 + tu\;\xi_1\cdot\xi_2\;\xi_3\cdot\xi_4 + st\;\xi_2\cdot\xi_4\;\xi_1\cdot\xi_3) \tag{2.9}$$

Other than the factor in front $\frac{4\pi g^2}{\epsilon_y\Lambda^4}$ it is precisely the kinematic factor of the corresponding tree (and the one-loop) diagram. The parameter ϵ_y is infinitesimal and introduced to regulate the divergence of the amplitude with the geometry vertex operator inserted. One sees that by adjusting $\frac{1}{\epsilon_y\Lambda^4}$ one can absorb the one-loop divergence. One of the nice things about the result is that the computation does not produce any finite part: the only power of ϵ_y that appears is $\frac{1}{\epsilon_y}$. As a matter of fact in all the computations that we have performed so far it remains true. The next leading order vertex operator is

$$\begin{aligned}\pi V_{G,r_0^{-5}} =& \; -\frac{i}{4}(\sqrt{h}h^{ij} - \varepsilon^{ij})\partial_i X^+ H_0^{-5/4}\frac{H_0'}{r_0}\partial_j X^m X_0^n \; (S\gamma^{mn}S) \\ & +\frac{i}{4}(\sqrt{h}h^{ij} - \varepsilon^{ij})\partial_i X^+ H_0^{-7/4}\frac{H_0'}{r_0}\partial_j X^u X_0^n \; (S\gamma^{un}S)\end{aligned} \tag{2.10}$$

At this order the amplitude turn out to vanish,

$$\langle V_s V_s V_s V_s \; V_{G,r_0^{-5}} \rangle = 0 \tag{2.11}$$

In the third leading order the geometry vertex operator is

$$\begin{aligned}V_{G,r_0^{-6}} =& \; -\frac{i\partial_i X^+}{2p^+}(\sqrt{h}\;h^{ij} - \varepsilon^{ij})\Big[X^n X^n \; S\partial_j S + \partial_j X^u X^n \; (S\gamma^{un}S)) \\ & \qquad\qquad -\partial_j X^m X^n \; (S\gamma^{mn}S)\Big] \\ & -\frac{1}{192}\sqrt{h}h^{ij}\frac{\partial_i X^+ \partial_j X^+}{(p^+)^2}\;\{(S\gamma^{au}S)(S\gamma^{au}S) - (S\gamma^{ab}S)(S\gamma^{ab}S)\}\end{aligned} \tag{2.12}$$

With increasing number of the fields the computation becomes quickly complicated even for the machine computing. Although we have not entirely completed computation we have carried out some of the correlators. For example we have checked that

$$\langle XXXX \; V_{G,r_0^{-6}} \rangle = 0 \tag{2.13}$$

There are several other corrrelators that we have checked. Based on the computations so far we expect that the correlator at this order will vanish, $\langle V_s V_s V_s V_s \; V_{G,r_0^{-6}}\rangle = 0$. We mention the reason for the expectation in the conclusion.

2.2 Vector multiplet scattering

A similar pattern is found in the case of the vector scattering: the leading order terms in V_G produces the desired kinematic structure and the higher order terms yield vanishing results. Recall the kinematic structure of the tree level scattering without V_G,

$$\begin{aligned} K = \; & -\frac{1}{4}(st\;\zeta_1\cdot\zeta_3\;\zeta_2\cdot\zeta_4 + su\;\zeta_2\cdot\zeta_3\;\zeta_1\cdot\zeta_4 + tu\;\zeta_1\cdot\zeta_2\;\zeta_3\cdot\zeta_4) \\ & +\frac{1}{2}s(\zeta_1\cdot k_4\;\zeta_3\cdot k_2\;\zeta_2\cdot\zeta_4 + \zeta_2\cdot k_3\;\zeta_4\cdot k_1\;\zeta_1\cdot\zeta_3 \\ & \quad +\zeta_1\cdot k_3\;\zeta_4\cdot k_2\;\zeta_2\cdot\zeta_3 + \zeta_2\cdot k_4\;\zeta_3\cdot k_1\;\zeta_1\cdot\zeta_4) \\ & +\frac{1}{2}t(\zeta_2\cdot k_1\;\zeta_4\cdot k_3\;\zeta_3\cdot\zeta_1 + \zeta_3\cdot k_4\;\zeta_1\cdot k_2\;\zeta_2\cdot\zeta_4 \\ & \quad +\zeta_2\cdot k_4\;\zeta_1\cdot k_3\;\zeta_3\cdot\zeta_4 + \zeta_3\cdot k_1\;\zeta_4\cdot k_2\;\zeta_2\cdot\zeta_1) \\ & +\frac{1}{2}u(\zeta_1\cdot k_2\;\zeta_4\cdot k_3\;\zeta_3\cdot\zeta_2 + \zeta_3\cdot k_4\;\zeta_2\cdot k_1\;\zeta_1\cdot\zeta_4 \\ & \quad +\zeta_1\cdot k_4\;\zeta_2\cdot k_3\;\zeta_3\cdot\zeta_4 + \zeta_3\cdot k_2\;\zeta_4\cdot k_1\;\zeta_1\cdot\zeta_2) \end{aligned} \tag{2.14}$$

One can show that

$$\langle V_g V_g V_g V_g \; V_{G,r_0^{-4}}\rangle = \frac{4\pi g_s}{\epsilon_y \Lambda^4}\; K \tag{2.15}$$

In sec 3 we illustrate the computations explicitly working out the coefficients of all $\zeta\cdot\zeta\;\zeta\cdot\zeta$-terms and a few $\zeta\cdot k\;\zeta\cdot k\;\zeta\cdot\zeta$-terms. The next order geometry vertex operator yields vanishing expression as in the scalar case,

$$\langle V_g V_g V_g V_g \; V_{G,r_0^{-5}}\rangle = 0 \tag{2.16}$$

The results of the two subsections above verify the conjecture at the first two leading orders. Let's compare the results with the one loop divergence. The one loop divergence in each case comes with a diverging factor

$$\int_\epsilon \frac{1}{y^2} \sim \frac{1}{\epsilon} \tag{2.17}$$

This implies a relation between ϵ, ϵ_y and Λ. Up to an immaterial numerical factor it is

$$\frac{1}{\epsilon_y \Lambda^4} = \frac{1}{\epsilon} \tag{2.18}$$

We now turn to the actual derivation of the results presented in this section.

JHEP08(2008)026

3. One loop divergence cancellation

An M-point amplitude in general is given by[5]

$$A_M = \int d\mu \left\langle \prod_{i=1}^{M} V(k_i) \right\rangle \tag{3.1}$$

The measure $d\mu$ is

$$d\mu = |(x_1 - x_2)(x_1 - x_M)(x_2 - x_M)| \int dx_3 \dots dx_{M-1} \prod_{1}^{M-1} \theta(x_r - x_{r+1}) \tag{3.2}$$

To remove the divergence we proposed [4] to consider

$$A_M = \int_0^1 dx \int_{x_1}^{\infty} dy \left\langle \prod_{i=1}^{M} V(k_i)\, V_G(y) \right\rangle \tag{3.3}$$

where $V(k_i)$ denotes an external state and V_G the geometry vertex operator. We have chosen the location of states such that $x_4 < x_3 < x_2 < x_1 < y$ with $x_4 = 0, x_3 = x, x_2 = 1$. This is a natural choice in light of the view that $V_G|0>$ represents a some kind of asymptotic state. At the end of each computation we take $x_1 \to \infty$. Since the measure gives the factor x_1^2 one can keep only the terms that comes with $\frac{1}{x_1^2}$ when computing $\left\langle \prod_{i=1}^{M} V(k_i)\, V_G(y) \right\rangle$. To regulate the divergence that occurs when $y \to x_1$ the y-integral range is adjusted to

$$\int_{x_1+\epsilon_y}^{\infty} dy \tag{3.4}$$

We will focus on the four point amplitudes. As for the three point, one loop, with or without the geometry vertex operator, vanishes due to the index structures. One of the expansion parameters is taken to be q,

$$q = \frac{Q}{r_0^4} \quad \text{with} \;\; Q = 4\pi g^2 \alpha'^2 \tag{3.5}$$

Note the following to keep the same orders of the expansion parameters

$$\partial X^+ = l^2 p^+, \quad H_0 = 1 + q, \quad H_0' = -4\frac{q}{r_0} \tag{3.6}$$

Introducing a dimensionless constant Λ we measure the norm of r_0 in terms of the string constant α'

$$r_0^4 = \Lambda^4\, \alpha'^2 \tag{3.7}$$

so that

$$q = \frac{4\pi g^2}{\Lambda^4} \tag{3.8}$$

[5] For a review see [11, 12]

JHEP08(2008)026

The parameter N that represents the number of branes will appear through the Chan-Paton procedure. We take

$$\Lambda, g \tag{3.9}$$

as the expansion parameters for our perturbative analysis. The bosonic and the fermionic propagators are respectively

$$\begin{aligned} \langle X^i X^j \rangle &= -2\alpha' \eta^{ij} \ln |x - x'| \\ \langle S_1^{a_1} S_1^{a_2} \rangle &= \frac{\delta^{a_1 a_2}}{x_1 - x_2} \end{aligned} \tag{3.10}$$

In the computations below we wick-rotate not only the world-sheet parameter τ but also σ. The latter is implied by T-duality. The same Wick rotation was used in the previous work [4]. It is useful to note that

$$\mathrm{Tr}\, \gamma^{u_1 v_1} \gamma^{u_2 v_2} = -8(\delta_{u_1 u_2} \delta_{v_1 v_2} - \delta_{u_1 v_2} \delta_{u_2 v_1}), \tag{3.11}$$

The γ's here are 8 by 8 matrices. One can easily check that

$$\langle R^{u_1 v_1} R^{u_2 v_2} \rangle = -\frac{(\delta_{u_1 u_2} \delta_{v_1 v_2} - \delta_{u_1 v_2} \delta_{u_2 v_1})}{(x_1 - x_2)^2} \tag{3.12}$$

and

$$\begin{aligned} &\langle R^{u_1 v_1}(x_1) R^{u_2 v_2}(x_2) R^{u_3 v_3}(x_3) \rangle \\ &\qquad = -\frac{1}{x_{12} x_{23} x_{13}} \, (\delta_{u_2 u_3} \delta_{u_1 v_2} \delta_{v_1 v_3} - \delta_{u_2 u_3} \delta_{u_1 v_3} \delta_{v_1 v_2} - \delta_{u_2 v_3} \delta_{u_1 v_2} \delta_{v_1 u_3} \\ &\qquad\qquad + \delta_{u_2 v_3} \delta_{u_1 u_3} \delta_{v_1 v_2} - \delta_{u_3 v_2} \delta_{u_1 u_2} \delta_{v_1 v_3} + \delta_{u_3 v_2} \delta_{u_1 v_3} \delta_{u_2 v_1} \\ &\qquad\qquad + \delta_{v_2 v_3} \delta_{u_1 u_2} \delta_{v_1 u_3} - \delta_{v_2 v_3} \delta_{u_1 u_3} \delta_{u_2 v_1}) \end{aligned} \tag{3.13}$$

The product of four R's can be similarly computed. The result is rather long so we do not present it here but refer to [4]. In many intermediate steps of the computations below momentum conservation is used. For example in some of the correlators the leading order term comes with $\frac{1}{x_1}$. This would lead to a divergent result since only a factor of x_1^2 is present in the integration measure. Often the term gets killed by momentum conservation if not by its index structure. We keep $\alpha', l(\equiv \sqrt{2\alpha'})$ in some places but in others we have used their explicit values,

$$\alpha' = \frac{1}{2}, \quad l = 1 \tag{3.14}$$

3.1 Scalar scattering case

For convenience we record the explicit form of the product of four scalar vertex operators

$$\begin{aligned} &V_s^{m_1}(x_1) V_s^{m_2}(x_2) V_s^{m_3}(x_3) V_s^{m_4}(x_4) \\ &\qquad = X'^{m_1} X'^{m_2} X'^{m_3} X'^{m_4} + l^8 R^{m_1 v_1} k_1^{v_1} R^{m_2 v_2} k_2^{v_2} R^{m_3 v_3} k_3^{v_3} R^{m_4 v_4} k_4^{v_4} \end{aligned} \tag{3.15}$$

JHEP08(2008)026

$$
\begin{aligned}
&-l^2\left[X'^{m_1}X'^{m_2}X'^{m_3}R^{m_4v_4}k_4^{v_4}+X'^{m_1}X'^{m_2}X'^{m_4}R^{m_3v_3}k_4^{v_3}\right.\\
&\qquad\left.+X'^{m_1}X'^{m_3}X'^{m_4}R^{m_2v_2}k_2^{v_2}+X'^{m_2}X'^{m_3}X'^{m_4}R^{m_1v_1}k_1^{v_1}\right]\\
&+l^4\left[X'^{m_1}X'^{m_2}R^{m_3v_3}k_3^{v_3}R^{m_4v_4}k_4^{v_4}+X'^{m_3}X'^{m_4}R^{m_1v_1}k_1^{v_1}R^{m_2v_2}k_2^{v_2}\right.\\
&\qquad+X'^{m_1}X'^{m_4}R^{m_2v_2}k_2^{v_2}R^{m_3v_3}k_3^{v_3}+X'^{m_1}X'^{m_3}R^{m_2v_2}k_2^{v_2}R^{m_4v_4}k_4^{v_4}\\
&\qquad\left.+X'^{m_2}X'^{m_3}R^{m_1v_1}k_1^{v_1}R^{m_4v_4}k_4^{v_4}+X'^{m_2}X'^{m_4}R^{m_1v_1}k_1^{v_1}R^{m_3v_3}k_3^{v_3}\right]\\
&-l^6\left[X'^{m_1}R^{m_2v_2}k_2^{v_2}R^{m_3v_3}k_3^{v_3}R^{m_4v_4}k_4^{v_4}+X'^{m_2}R^{m_1v_1}k_1^{v_1}R^{m_3v_3}k_3^{v_3}R^{m_4v_4}k_4^{v_4}\right.\\
&\qquad\left.+X'^{m_3}R^{m_1v_1}k_1^{v_1}R^{m_2v_2}k_2^{v_2}R^{m_4v_4}k_4^{v_4}+X'^{m_4}R^{m_1v_1}k_1^{v_1}R^{m_2v_2}k_2^{v_2}R^{m_3v_3}k_3^{v_3}\right]
\end{aligned}
$$

The form is appropriate before the Wick rotation which will be taken into account in each individual computation below. With it we multiply the geometry vertex operator at each order, such as $V_{G,r_0^{-4}}, V_{G,r_0^{-5}}$ etc, and compute the resulting correlator. Each level geometry vertex operator has several terms: we compute them one by one and put the results together at the end. A pattern emerges on how the desired kinematic structure arises: only a first few leading terms are responsible for the structure with the higher order terms yielding vanishing results.

3.1.1 Leading order computation

The leading vertex operator is given by

$$
\begin{aligned}
\pi V_{G,r_0^{-4}} &\simeq \frac{q}{4}\sqrt{h}\,h^{ij}\left(\partial_i X^u\partial_j X^v\eta_{uv}-\partial_i X^m\partial_j X^n\eta_{mn}\right)+\frac{il^2 q}{4}(\sqrt{h}\,h^{0j}+\varepsilon^{0j})(S\partial_j S)\\
&\simeq \frac{q}{4}\left(-\partial_\sigma X^m\partial_\sigma X^m-\partial_\tau X^u\partial_\tau X^u+il^2\left(-S\partial_\tau S-S\partial_\sigma S\right)\right) \qquad (3.16)
\end{aligned}
$$

When one goes from the first to second one drops certain bosonic terms because of $\sigma=0$. With the first correlator

$$
\langle V_s^{m_1}(x_1)V_s^{m_2}(x_2)V_s^{m_3}(x_3)V_s^{m_4}(x_4)\;\partial_i X^m\partial_j X^n\eta_{mn}\rangle,
$$

certain terms drop either because of the dimensional regularization or/and they contain an odd number of X^m fields:

$$
\begin{aligned}
&\langle V_s^{m_1}(x_1)V_s^{m_2}(x_2)V_s^{m_3}(x_3)V_s^{m_4}(x_4)\;\partial_i X^m(y)\partial_j X^n(y)\eta_{mn}\rangle\\
&\quad=\langle X'^{m_1}X'^{m_2}X'^{m_3}X'^{m_4}\;\partial_i X^m\partial_j X^n\eta_{mn}\rangle\\
&\qquad-l^4\langle\left[X'^{m_1}X'^{m_2}R^{m_3v_3}k_3^{v_3}R^{m_4v_4}k_4^{v_4}+X'^{m_3}X'^{m_4}R^{m_1v_1}k_1^{v_1}R^{m_2v_2}k_2^{v_2}\right.\\
&\qquad\quad+X'^{m_1}X'^{m_4}R^{m_2v_2}k_2^{v_2}R^{m_3v_3}k_3^{v_3}+X'^{m_1}X'^{m_3}R^{m_2v_2}k_2^{v_2}R^{m_4v_4}k_4^{v_4} \qquad (3.17)\\
&\qquad\quad\left.+X'^{m_2}X'^{m_3}R^{m_1v_1}k_1^{v_1}R^{m_4v_4}k_4^{v_4}+X'^{m_2}X'^{m_4}R^{m_1v_1}k_1^{v_1}R^{m_3v_3}k_3^{v_3}\right]\;\partial_i X^m\partial_j X^n\eta_{mn}\rangle
\end{aligned}
$$

The signs of l^4-terms have been flipped by Wick rotation in the σ direction, which is implied by T-duality. Combining the two contributions one gets after collecting terms of the

JHEP08(2008)026

same types

$$\int_0^1 dx \int_{x_1+\epsilon_y}^{\infty} dy \, \langle V_s^{m_1}(x_1)V_s^{m_2}(x_2)V_s^{m_3}(x_3)V_s^{m_4}(x_4) \, \partial_i X^m(y)\partial_j X^n(y)\eta_{mn}\rangle$$

$$= \int_0^1 dx \left[-\frac{16}{\epsilon_4}\left(\xi_1\cdot\xi_4\,\xi_2\cdot\xi_3\,\frac{1}{(-1+x)^2} + \xi_1\cdot\xi_3\,\xi_2\cdot\xi_4 + \frac{\xi_1\cdot\xi_2\,\xi_3\cdot\xi_4}{x^2}\right)\alpha'^3 \right.$$
$$\left. -\frac{16}{\epsilon_4}\left(\frac{t\,\xi_1\cdot\xi_4\,\xi_2\cdot\xi_3}{(-1+x)^2} + u\,\xi_1\cdot\xi_3\,\xi_2\cdot\xi_4 + \frac{s\,\xi_1\cdot\xi_2\,\xi_3\cdot\xi_4}{x^2}\right)\alpha'^4 \right] \tag{3.18}$$

After performing the x-integration it reproduces the tree result of the four point amplitude without the geometry vertex operator,

$$\frac{1}{\epsilon_y}\,\frac{1}{4}(su\;\xi_1\cdot\xi_4\;\xi_2\cdot\xi_3 + tu\;\xi_1\cdot\xi_2\;\xi_3\cdot\xi_4 + st\;\xi_2\cdot\xi_4\;\xi_1\cdot\xi_3) \tag{3.19}$$

The next correlator to consider is

$$\langle V_s^{m_1}(x_1)V_s^{m_2}(x_2)V_s^{m_3}(x_3)V_s^{m_4}(x_4)\;\partial_i X^u\partial_j X^v\eta_{uv}\rangle$$

Here again certain terms drop trivially either because of the dimensional regularization or/and because they contain an odd number of X^m fields:

$$\langle V_s^{m_1}(x_1)V_s^{m_2}(x_2)V_s^{m_3}(x_3)V_s^{m_4}(x_4)\;\partial_i X^u(y)\partial_j X^v(y)\eta_{uv}\rangle$$
$$= \langle X'^{m_1}X'^{m_2}X'^{m_3}X'^{m_4}\;\partial_i X^u\partial_j X^v\eta_{uv}\rangle$$
$$+l^8\langle R^{m_1v_1}k_1^{v_1}R^{m_2v_2}k_2^{v_2}R^{m_3v_3}k_3^{v_3}R^{m_4v_4}k_4^{v_4}\;\partial_i X^u\partial_j X^v\eta_{uv}\rangle$$
$$-l^4\langle\big[X'^{m_1}X'^{m_2}R^{m_3v_3}k_3^{v_3}R^{m_4v_4}k_4^{v_4} + X'^{m_3}X'^{m_4}R^{m_1v_1}k_1^{v_1}R^{m_2v_2}k_2^{v_2}$$
$$+X'^{m_1}X'^{m_4}R^{m_2v_2}k_2^{v_2}R^{m_3v_3}k_3^{v_3} + X'^{m_1}X'^{m_3}R^{m_2v_2}k_2^{v_2}R^{m_4v_4}k_4^{v_4} \tag{3.20}$$
$$+X'^{m_2}X'^{m_3}R^{m_1v_1}k_1^{v_1}R^{m_4v_4}k_4^{v_4} + X'^{m_2}X'^{m_4}R^{m_1v_1}k_1^{v_1}R^{m_3v_3}k_3^{v_3}\big]\;\partial_i X^u\partial_j X^v\eta_{uv}\rangle$$

After some algebra one can show that

$$\langle V_s^{m_1}(x_1)V_s^{m_2}(x_2)V_s^{m_3}(x_3)V_s^{m_4}(x_4)\;\partial_i X^u(y)\partial_j X^v(y)\eta_{uv}\rangle \;\sim \frac{1}{x_1^3} \tag{3.21}$$

It vanishes as $x_1 \to \infty$ since it still goes $\sim \frac{1}{x_1}$ even after taking the measure into account. As a matter of fact many terms in the higher-order V_G seem to vanish for the same reason. The last correlator which is with $S(y)\partial S(y)$ vanishes due to the fermionic equation of motion. This completes the proof that at the leading order in r_0-expansion the geometry vertex operator does produce the correct structure to cancel the one-loop divergence.

3.1.2 Next leading order computation

In the next leading order the geometry vertex operator is[6]

$$\pi V_{G,r_0^{-5}} = -\frac{i}{4}(\sqrt{h}h^{ij}-\varepsilon^{ij})\partial_i X^+ H_0^{-5/4}\frac{H_0'}{r_0}\partial_j X^m X_0^n\;(S\gamma^{mn}S)$$
$$+\frac{i}{4}(\sqrt{h}h^{ij}-\varepsilon^{ij})\partial_i X^+ H_0^{-7/4}\frac{H_0'}{r_0}\partial_j X^u X_0^n\;(S\gamma^{un}S) \tag{3.22}$$

[6] There are terms that are of the form $\partial X\partial XX$ that come from the first line of (2.1) after expanding $H^{\pm 1/2}$. They trivially vanish due to the dimensional regularization and/or their index structures.

JHEP08(2008)026

As before several terms drop trivially

$$\begin{aligned}
&\langle V_s^{m_1}(x_1)V_s^{m_2}(x_2)V_s^{m_3}(x_3)V_s^{m_4}(x_4)\ \partial_j X^m S\gamma^{mn}S\rangle X_0^n \\
&\quad= \langle(-l^2\left[X'^{m_1}X'^{m_2}X'^{m_3}R^{m_4v_4}k_4^{v_4}+X'^{m_1}X'^{m_2}X'^{m_4}R^{m_3v_3}k_4^{v_3}\right.\\
&\qquad\qquad \left.+X'^{m_1}X'^{m_3}X'^{m_4}R^{m_2v_2}k_2^{v_2}+X'^{m_2}X'^{m_3}X'^{m_4}R^{m_1v_1}k_1^{v_1}\right]\\
&\qquad -l^6\left[X'^{m_1}R^{m_2v_2}k_2^{v_2}R^{m_3v_3}k_3^{v_3}R^{m_4v_4}k_4^{v_4}+X'^{m_2}R^{m_1v_1}k_1^{v_1}R^{m_3v_3}k_3^{v_3}R^{m_4v_4}k_4^{v_4}\right.\\
&\qquad\qquad \left.+X'^{m_3}R^{m_1v_1}k_1^{v_1}R^{m_2v_2}k_2^{v_2}R^{m_4v_4}k_4^{v_4}+X'^{m_4}R^{m_1v_1}k_1^{v_1}R^{m_2v_2}k_2^{v_2}R^{m_3v_3}k_3^{v_3}\right])\\
&\qquad \partial_j X^m S\gamma^{mn}S\rangle X_0^n\\
&\quad= \langle(-l^6\left[X'^{m_1}R^{m_2v_2}k_2^{v_2}R^{m_3v_3}k_3^{v_3}R^{m_4v_4}k_4^{v_4}+X'^{m_2}R^{m_1v_1}k_1^{v_1}R^{m_3v_3}k_3^{v_3}R^{m_4v_4}k_4^{v_4}\right.\\
&\qquad\qquad \left.+X'^{m_3}R^{m_1v_1}k_1^{v_1}R^{m_2v_2}k_2^{v_2}R^{m_4v_4}k_4^{v_4}+X'^{m_4}R^{m_1v_1}k_1^{v_1}R^{m_2v_2}k_2^{v_2}R^{m_3v_3}k_3^{v_3}\right])\\
&\qquad \partial_j X^m S\gamma^{mn}S\rangle X_0^n
\end{aligned} \tag{3.23}$$

One can see that the l^2-terms vanish due to the index structure as follows. For example consider $< R^{m_1v_1}\ S\gamma^{mn}S >\sim(\delta_{m_1m}\delta_{v_1n}-\delta_{m_1n}\delta_{v_1m})=0$. (Recall that the m or n indices run in the transverse directions whereas the u or v in the longitudinal space.) The l^6-terms also vanish for the same reason. Therefore at this order one gets

$$\langle V_sV_sV_sV_s\ V_{G,r_0^{-5}})\rangle = 0 \tag{3.24}$$

3.2 Vector scattering case

The explicit form of the product of four vector vertex operators is

$$\begin{aligned}
&V_g^{u_1}(x_1)V_g^{u_2}(x_2)V_g^{u_3}(x_3)V_g^{u_4}(x_4)\\
&\quad= \dot X^{u_1}\dot X^{u_2}\dot X^{u_3}\dot X^{u_4}+l^8R^{u_1v_1}k_1^{v_1}R^{u_2v_2}k_2^{v_2}R^{u_3v_3}k_3^{v_3}R^{u_4v_4}k_4^{v_4}\\
&\qquad -l^2\Big[\dot X^{u_1}\dot X^{u_2}\dot X^{u_3}R^{u_4v_4}k_4^{v_4}+\dot X^{u_1}\dot X^{u_2}\dot X^{u_4}R^{u_3v_3}k_4^{v_3}\\
&\qquad\qquad +\dot X^{u_1}\dot X^{u_3}\dot X^{u_4}R^{u_2v_2}k_2^{v_2}+\dot X^{u_2}\dot X^{u_3}\dot X^{u_4}R^{u_1v_1}k_1^{v_1}\Big]\\
&\qquad +l^4\Big[\dot X^{u_1}\dot X^{u_2}R^{u_3v_3}k_3^{v_3}R^{u_4v_4}k_4^{v_4}+\dot X^{u_3}\dot X^{u_4}R^{u_1v_1}k_1^{v_1}R^{u_2v_2}k_2^{v_2}\\
&\qquad\qquad +\dot X^{u_1}\dot X^{u_4}R^{u_2v_2}k_2^{v_2}R^{u_3v_3}k_3^{v_3}+\dot X^{u_1}\dot X^{u_3}R^{u_2v_2}k_2^{v_2}R^{u_4v_4}k_4^{v_4}\\
&\qquad\qquad +\dot X^{u_2}\dot X^{u_3}R^{u_1v_1}k_1^{v_1}R^{u_4v_4}k_4^{v_4}+\dot X^{u_2}\dot X^{u_4}R^{u_1v_1}k_1^{v_1}R^{u_3v_3}k_3^{v_3}\Big]\\
&\qquad -l^6\Big[\dot X^{u_1}R^{u_2v_2}k_2^{v_2}R^{u_3v_3}k_3^{v_3}R^{u_4v_4}k_4^{v_4}+\dot X^{u_2}R^{u_1v_1}k_1^{v_1}R^{u_3v_3}k_3^{v_3}R^{u_4v_4}k_4^{v_4}\\
&\qquad\qquad +\dot X^{u_3}R^{u_1v_1}k_1^{v_1}R^{u_2v_2}k_2^{v_2}R^{u_4v_4}k_4^{v_4}+\dot X^{u_4}R^{u_1v_1}k_1^{v_1}R^{u_2v_2}k_2^{v_2}R^{u_3v_3}k_3^{v_3}\Big]
\end{aligned} \tag{3.25}$$

which is before the Wick rotation.

3.2.1 Leading order computation

Since the large r_0-expansion is in terms of the transverse coordinates X^m and the transverse and the longitudinal coordinates do not contract each other one does not have to perform the expansion when considering purely vector state scattering.[7] To cancel the divergence

[7] This is true for the pure gauge boson scattering. Once one puts the gaugino state it will not be so in general since it contains X' as well as $\dot X$.

JHEP08(2008)026

of the four vector scattering the relevant terms of the vertex operator V_G are

$$\begin{aligned}
\pi V_G \Rightarrow \quad & -\frac{1}{2}\sqrt{h}\, h^{ij}\left(\partial_i X^u \partial_j X^v \eta_{uv}(H_0^{-1/2}-1)\right) \\
&+\frac{1}{2p^+}\Big\{ -2i(\sqrt{h}\, h^{ij}-\varepsilon^{ij})\partial_i X^+ (H_0^{-1/4}-1)(S\partial_j S) \\
&\qquad +\frac{i}{4}(\sqrt{h}h^{ij}-\varepsilon^{ij})\partial_i X^+ H_0^{-7/4}\frac{H_0'}{r_0}\partial_j X^u X_0^m\,(S\gamma^{um}S)\Big\} \\
&+\frac{1}{4(p^+)^2}\sqrt{h}h^{ij}\partial_i X^+ \partial_j X^+\, H_0^{-1/2}\Big\{ -\frac{17}{1536}\kappa_{10}(S\gamma^{uv}S)(S\gamma^{uv}S) \\
&\qquad +\left[\frac{43}{768}\kappa_{10}+\frac{1}{192}\kappa_{20}\right](S\gamma^{au}S)(S\gamma^{au}S) \\
&\qquad -\left[\frac{1}{192}\kappa_{20}+\frac{1}{128}\kappa_{10}\right](S\gamma^{ab}S)(S\gamma^{ab}S) \\
&\qquad +X_0^a X_0^b\frac{1}{r_0^2}\left[\frac{31}{768}\kappa_{10}-\frac{1}{32}\kappa_{20}\right](S\gamma^{au}S)(S\gamma^{bu}S) \\
&\qquad +X_0^a X_0^b\frac{1}{r_0^2}\left[+\frac{1}{32}\kappa_{20}+\frac{29}{384}\kappa_{10}\right](S\gamma^{ac}S)(S\gamma^{bc}S)\Big\}
\end{aligned} \tag{3.26}$$

A few of the S-quadratic terms have been dropped because it does not make any contribution in the dimensional regularization. The leading order operator is

$$\pi V_{G,r_0^{-4}} \;\Rightarrow \frac{q}{4}\left(-\partial_\tau X^u \partial_\tau X^u + il^2\left(-S\partial_\tau S - S\partial_\sigma S\right)\right) \tag{3.27}$$

As for the correlators with $(-S\partial_\tau S - S\partial_\sigma S)$ it vanishes because of the fermionic field equation. Some of the terms are trivially zero. The $\langle RRRR\partial X\partial X\rangle$ goes as $1/x_1^4$ so it lead to a vanishing result, so one can compute

$$\begin{aligned}
\langle\Big(&\dot X^{u_1}\dot X^{u_2}\dot X^{u_3}\dot X^{u_4} \\
&+l^4\Big[\dot X^{u_1}\dot X^{u_2}R^{u_3v_3}k_3^{v_3}R^{u_4v_4}k_4^{v_4}+\dot X^{u_3}\dot X^{u_4}R^{u_1v_1}k_1^{v_1}R^{u_2v_2}k_2^{v_2} \\
&\quad+\dot X^{u_1}\dot X^{u_4}R^{u_2v_2}k_2^{v_2}R^{u_3v_3}k_3^{v_3}+\dot X^{u_1}\dot X^{u_3}R^{u_2v_2}k_2^{v_2}R^{u_4v_4}k_4^{v_4} \\
&\quad+\dot X^{u_2}\dot X^{u_3}R^{u_1v_1}k_1^{v_1}R^{u_4v_4}k_4^{v_4}+\dot X^{u_2}\dot X^{u_4}R^{u_1v_1}k_1^{v_1}R^{u_3v_3}k_3^{v_3}\Big] \\
&-l^6\Big[\dot X^{u_1}R^{u_2v_2}k_2^{v_2}R^{u_3v_3}k_3^{v_3}R^{u_4v_4}k_4^{v_4}+\dot X^{u_2}R^{u_1v_1}k_1^{v_1}R^{u_3v_3}k_3^{v_3}R^{u_4v_4}k_4^{v_4} \\
&\quad+\dot X^{u_3}R^{u_1v_1}k_1^{v_1}R^{u_2v_2}k_2^{v_2}R^{u_4v_4}k_4^{v_4}+\dot X^{u_4}R^{u_1v_1}k_1^{v_1}R^{u_2v_2}k_2^{v_2}R^{u_3v_3}k_3^{v_3}\Big]\Big) \\
&\partial_i X^u\partial_j X^v\eta_{uv}\rangle
\end{aligned} \tag{3.28}$$

The task is to reproduce the kinematic structure given in (2.14). We take a few examples to illustrate how things work. As for the coefficient of $\zeta_1\cdot\zeta_2\,\zeta_3\cdot\zeta_4$ only $XXXX\partial X\partial X$ and $XXRR\partial X\partial X$ contribute and one gets

$$\frac{\zeta_1\cdot\zeta_2\,\zeta_3\cdot\zeta_4}{x^2}\left(-\frac{16}{\epsilon_4}\,\alpha'^3-\frac{4}{\epsilon_4}\,\alpha'^2\right) \tag{3.29}$$

JHEP08(2008)026

Note that we sometimes use $\alpha' = 1/2$ here and there so the powers of α' is not systematic. After the x-integration one gets the expected result. Let's consider an example of the form $\zeta \cdot k\, \zeta \cdot k\, \zeta \cdot \zeta$ The coefficient of $\zeta_1 \cdot \zeta_2$ comes from $XXXX\partial X\partial X$ and $XXRR\partial X\partial X$:

$$32\left(\frac{k_2\cdot\zeta_3\, k_2\cdot\zeta_4}{-1+x}+\frac{k_2\cdot\zeta_3\, k_3\cdot\zeta_4}{(-1+x)\,x}+\frac{k_2\cdot\zeta_4\, k_4\cdot\zeta_3}{x}+\frac{k_3\cdot\zeta_4\, k_4\cdot\zeta_3}{x^2}\right)\frac{\alpha'^4}{\epsilon_4}$$
$$-\frac{8\,\zeta_3\cdot k_4\,\zeta_4\cdot k_3\,\alpha'^2}{x^2\,\epsilon_4} \tag{3.30}$$

which leads, after the x-integration, to the correct result of

$$u\,\zeta_3\cdot k_2\,\zeta_4\cdot k_1+t\,\zeta_3\cdot k_1\,\zeta_4\cdot k_2 \tag{3.31}$$

Even for the same $\zeta \cdot k\, \zeta \cdot k\, \zeta \cdot \zeta$-type terms the conspiracy between the intermediate terms can be different as can be seen in the computation of $\zeta_2 \cdot \zeta_4$-term. Here all three different type of terms contribute.

$$\langle XXXX\partial X\partial X\rangle \Rightarrow \frac{32}{\epsilon_4}\left(\frac{-\,\zeta_1\cdot k_2\,\zeta_3\cdot k_2}{-1+x}-\frac{x\,\zeta_1\cdot k_3\,\zeta_3\cdot k_2}{-1+x}-\frac{\zeta_1\cdot k_2\,\zeta_3\cdot k_4}{-1+x}\right.$$
$$\left.+\frac{\zeta_1\cdot k_2\,\zeta_3\cdot k_4}{(-1+x)\,x}-\zeta_1\cdot k_3\,\zeta_3\cdot k_4\right)\alpha'^4$$
$$-\langle XXRR\partial X\partial X\rangle \Rightarrow -\frac{\alpha'^3}{\epsilon_4}\left(\frac{8\,u\,\zeta_1\cdot k_2\,\zeta_3\cdot k_2}{-1+x}+\frac{8\,u\,x\,\zeta_1\cdot k_3\,\zeta_3\cdot k_2}{-1+x}+\frac{8\,u\,\zeta_1\cdot k_2\,\zeta_3\cdot k_4}{-1+x}\right.$$
$$\left.-\frac{8\,u\,\zeta_1\cdot k_2\,\zeta_3\cdot k_4}{(-1+x)\,x}-\frac{8\,u\,\zeta_1\cdot k_3\,\zeta_3\cdot k_4}{-1+x}+\frac{8\,u\,x\,\zeta_1\cdot k_3\,\zeta_3\cdot k_4}{-1+x}\right)$$
$$\langle XRRR\partial X\partial X\rangle \Rightarrow \frac{1}{\epsilon_4}\left(\frac{-i\,s\,\zeta_1\cdot k_2\,\zeta_3\cdot k_2}{(-1+x)\,x}-\frac{i\,s\,\zeta_1\cdot k_3\,\zeta_3\cdot k_2}{-1+x}+\frac{i\,t\,\zeta_1\cdot k_2\,\zeta_3\cdot k_4}{(-1+x)\,x}\right.$$
$$\left.+\frac{i\,t\,\zeta_1\cdot k_3\,\zeta_3\cdot k_4}{-1+x}\right) \tag{3.32}$$

One can easily show by doing the x-integration that

$$\langle XXXX\partial X\partial X\rangle-\langle XXRR\partial X\partial X\rangle-i\langle XRRR\partial X\partial X\rangle$$
$$\Rightarrow s\,\zeta_1\cdot k_4\,\zeta_3\cdot k_2+t\,\zeta_1\cdot k_2\,\zeta_3\cdot k_4 \tag{3.33}$$

where the Wick rot has been taken into account. The result is as expected. Each correlator above produces many unwanted terms of different structure, i.e., more "mixed" types of terms. They are combined to cancel among themselves. The details go as follows.

$$\langle XXXX\partial X\partial X\rangle-\langle XXRR\partial X\partial X\rangle-i\langle XRRR\partial X\partial X\rangle$$
$$\Rightarrow\frac{1}{\epsilon_4}\left[\frac{2}{(-1+x)^2\,x^2}\left(\zeta_1\cdot k_2+x\,\zeta_1\cdot k_3\right)\left(-\zeta_2\cdot k_3-\zeta_2\cdot k_4+x\,\zeta_2\cdot k_4\right)\right.$$
$$\left.\times\left(x\,\zeta_3\cdot k_2-\zeta_3\cdot k_4+x\,\zeta_3\cdot k_4\right)\left(x\,\zeta_4\cdot k_2+\zeta_4\cdot k_3\right)\right]$$
$$-\frac{1}{\epsilon_4}\left[\frac{2}{(-1+x)^2\,x^2}\left(\zeta_1\cdot k_2+x\,\zeta_1\cdot k_3\right)\left(-\zeta_2\cdot k_3-\zeta_2\cdot k_4+x\,\zeta_2\cdot k_4\right)\right.$$

JHEP08(2008)026

$$\times (x\,\zeta_3\cdot k_2 - \zeta_3\cdot k_4 + x\,\zeta_3\cdot k_4)\,(x\,\zeta_4\cdot k_2 + \zeta_4\cdot k_3) + \frac{2\,(\zeta_1\cdot k_2 + x\,\zeta_1\cdot k_3)\,(\zeta_2\cdot k_3\,\zeta_3\cdot k_4\,\zeta_4\cdot k_2 - \zeta_2\cdot k_4\,\zeta_3\cdot k_2\,\zeta_4\cdot k_3)}{(-1+x)\,x}\Bigg] - \frac{i}{\epsilon_4}\left[\frac{(2i)\,(\zeta_1\cdot k_2 + x\,\zeta_1\cdot k_3)\,(\zeta_2\cdot k_3\,\zeta_3\cdot k_4\,\zeta_4\cdot k_2 - \zeta_2\cdot k_4\,\zeta_3\cdot k_2\,\zeta_4\cdot k_3)}{(-1+x)\,x}\right] = 0 \tag{3.34}$$

3.2.2 Next leading order computation

For the vector scattering first consider $\langle V_g V_g V_g V_g\ \partial_j X^m (S\gamma^{mn} S)\rangle X_0^n$. By careful inspection of the indices one can show that it vanishes,

$$\langle V_g V_g V_g V_g\ \partial_j X^m (S\gamma^{mn} S)\rangle X_0^n = 0 \tag{3.35}$$

The second term in the geometry vertex operator gives

$$\begin{aligned}
&\langle \Big(l^8 R^{u_1 v_1} k_1^{v_1} R^{u_2 v_2} k_2^{v_2} R^{u_3 v_3} k_3^{v_3} R^{u_4 v_4} k_4^{v_4} \\
&\quad + l^4 \Big[\dot{X}^{u_1} \dot{X}^{u_2} R^{u_3 v_3} k_3^{v_3} R^{u_4 v_4} k_4^{v_4} + \dot{X}^{u_3} \dot{X}^{u_4} R^{u_1 v_1} k_1^{v_1} R^{u_2 v_2} k_2^{v_2} \\
&\qquad + \dot{X}^{u_1} \dot{X}^{u_4} R^{u_2 v_2} k_2^{v_2} R^{u_3 v_3} k_3^{v_3} + \dot{X}^{u_1} \dot{X}^{u_3} R^{u_2 v_2} k_2^{v_2} R^{u_4 v_4} k_4^{v_4} \\
&\qquad + \dot{X}^{u_2} \dot{X}^{u_3} R^{u_1 v_1} k_1^{v_1} R^{u_4 v_4} k_4^{v_4} + \dot{X}^{u_2} \dot{X}^{u_4} R^{u_1 v_1} k_1^{v_1} R^{u_3 v_3} k_3^{v_3} \Big] \Big) \\
&\partial_j X^u\ (S\gamma^{un} S)\rangle X_0^n
\end{aligned} \tag{3.36}$$

By inspecting the index structures again it is not difficult to tell that the above correlators vanish: basically because all the indices are (u, v) except one which is n. Therefore at this order

$$\langle V_g V_g V_g V_g\ V_{G, r_0^{-5}})\rangle = 0 \tag{3.37}$$

4. Discussion and future directions

In this work we have shown[8] at the first two leading orders in the large-r_0 expansion that the counter vertex operator (2.1) does produce the required structure (without any extra unwanted terms) to absorb the one-loop divergence. It is, therefore, verification of the conjecture put forward in [4] at the specified orders. It is encouraging that it is possible to absorb the divergence within the pure open string frame-work. For one thing it is not very clear how to produce the open string kinematic factor using some kind of explicit closed string degrees of freedom. Also even within the open string frame-work it is a priori never guaranteed that the computation will yield the right and only the right types of terms.

As the order increases, more[9] terms in the geometry vertex operator become relevant. With each term the number of the intermediate terms in the computations of the correlator increases very quickly, i.e., factorially. We have examined several r_0^{-6}-order terms. We

[8] For a D0 brane or D1 brane it is necessary to consider the recoil effect that was discussed for example in [13].

[9] However, only the finite number of terms contribute as mentioned previously.

JHEP08(2008)026

illustrate the computations with the scalar scattering. The counter vertex operator at r_0^{-6}-order is given by

$$\pi V_{G,r_0^{-6}} \simeq \; = \frac{q}{r_0^2}\Bigg(-\frac{1}{2}\sqrt{h}\, h^{ij}\,(\partial_i X^u \partial_j X^u X^n X^n - \partial_i X^m \partial_j X^m\; X^n X^n) \\ -\frac{i\partial_i X^+}{2p^+}(\sqrt{h}\, h^{ij} - \varepsilon^{ij})\,[X^n X^n\; S\partial_j S + \partial_j X^u X^n\;(S\gamma^{un}S)) \\ -\partial_j X^m X^n\;(S\gamma^{mn}S)] \\ -\frac{1}{192}\sqrt{h}h^{ij}\frac{\partial_i X^+ \partial_j X^+}{(p^+)^2}\;\{(S\gamma^{au}S)(S\gamma^{au}S) - (S\gamma^{ab}S)(S\gamma^{ab}S)\}\Bigg) \quad (4.1)$$

One of the correlators that we have considered is

$$\frac{q}{r_0^2}\langle V_s^{m_1}(x_1)V_s^{m_2}(x_2)V_s^{m_3}(x_3)V_s^{m_4}(x_4)\;\partial_i X^m \partial_j X^n \eta_{mn} X^l X^l\rangle \\ = \frac{q}{r_0^2}\langle X'^{m_1} X'^{m_2} X'^{m_3} X'^{m_4}\;\partial_i X^m \partial_j X^m X^n X^n\rangle \quad (4.2)$$

The other terms drop due to the dimensional regularization. It turns out that the correlator vanishes

$$\langle X'^{m_1} X'^{m_2} X'^{m_3} X'^{m_4}\;\partial_i X^m \partial_j X^m X^n X^n\rangle = 0 \quad (4.3)$$

Therefore

$$\frac{q}{r_0^2}\langle V_s^{m_1}(x_1)V_s^{m_2}(x_2)V_s^{m_3}(x_3)V_s^{m_4}(x_4)\;\partial_i X^m \partial_j X^n \eta_{mn} X^l X^l\rangle = 0 \quad (4.4)$$

As a matter of fact it is not too difficult to check that none of the r_0^{-6}-terms yields a finite result,

$$\langle X'^{m_1} X'^{m_2} X'^{m_3} X'^{m_4}\; V_{G,r_0^{-6}}\rangle = 0 \quad (4.5)$$

Another correlator that we have checked is $\langle XXRR\,\partial X^u \partial X^u X^n X^n\rangle$. It also vanishes,

$$\langle XXRR\,\partial X^u \partial X^u X^n X^n\rangle = 0 \quad (4.6)$$

In the higher order computations it is often the high powers of $\frac{1}{x_1}$ that are responsible for the null result since higher order terms tend to come with higher powers of $\frac{1}{x_1}$. We expect with reasonable confidence that all the higher order terms will yield vanishing results because of this reason together with the index structures.

In the introduction we have shown that the flat space is unable to cancel the divergence due to a mismatch of a sign, which is correctly produced by the action in the curved space. Together with the results obtained in sec 3 we believe that it strongly supports the notion of the engineering of the D-brane geometry by open string loop effects. However, the fact that all the higher order correlators checked so far vanish makes role-less all the terms in V_G that are more composite than those in quadratic order in fields. It will be nice to see an example where that they do contribute. It is likely that the non-contribution of the

JHEP08(2008)026

higher order terms is a peculiar feature of the one-loop order. At higher loop orders we expect that the presence and inter-correlation of the cubic and more composite terms will be indispensable for the cancelation of the divergence. It is one of the near-future directions that we will pursue [14].[10] Another direction that we may pursue is the computation of the open string analogue of the anomalous dimension of N=4 SYM. It will be interesting to study whether the full open string computation can lead to a resolution of the three loop discrepancy.

Once we verify the conjecture with more examples of higher orders in r_0 or/and g one of the things that it establishes is the picture that the open string, starting out in a flat space, completes the theory toward the curved geometry. It will also imply the relevance of the open string even in the final form of AdS/CFT [16].[11] The resulting non-linear sigma model then will be analogous to the 1PI effective action in a quantum field theory. The connection to AdS geometry and to AdS/CFT can be seen through the effective field theory action, namely the DBI type action, along the line of the following logic [18, 4]. First apply an S-duality on the DBI action making the coupling constant flip, $g \to g' = \frac{1}{g}$. Then taking a $g \to 0$ limit brings two things. First, the D-brane geometry becomes an AdS space. Secondly, the limit allows one to write down the solution of the equations of motion of the DBI action in a particular form [20, 21], which, in turn, can be interpreted as a closed string action.

Finally a few comments on the relation with the Fischler-Susskind mechanism [6, 7] are in order. The very idea of the role of the geometry in the divergence cancelation is the same, in spirit, as that of the Fischler-Susskind mechanism. There are a few differences as well. First of all, it is the set-up of the computation, which in turn makes the interpretation of the geometry very different. In [6, 7] (a related discussion can be found in [19]) the geometry exists from the beginning as a fundamental object whereas in our construction we are proposing that it should be a secondary by-product of the flat-space loop effects. Secondly but perhaps more importantly it is the relevance of the presence of a D-brane and the transverse space. The analysis of [6, 7, 19] was carried out for a closed string/space-time filling brane case. Therefore there is no room for the transverse geometry. This is to be contrasted with the present case where have a Dp-brane with $p < 9$. Put in another way, we do not expect the D9 brane to have non-trivial geometry even in the higher order perturbation. Lastly, we note that the geometry that results from the analysis of [6, 7] is AdS/dS while in our case it is (or is expected to be) the full Dp-brane geometry before the S-duality that is mentioned in the previous paragraph.

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[10] For that purpose it may be useful to attempt the corresponding computation in the field theory context in an extension of the analysis that is initiated in [15].

[11] A related discussion can be found e.g., in [17].

JHEP08(2008)026

A. Derivation of geometry vertex operator

A.1 Derivation of the action

The GS action for a generic curved background was obtained by several different groups [8–10]. We use the action obtained by Sahakian since his result includes fermionic quartic terms which is the highest order, in the light-cone gauge, that can be present for a certain class of configurations. We narrow down to the terms that is relevant for the D3-brane geometry. The action that is zeroth order in the fermionic coordinates is given, in our conventions, by

$$S^{(0)} = -\int d^2\sigma \,\left[\frac{1}{2}\sqrt{h}\, h^{ij} V_i^{\tilde{A}} V_{\tilde{A}j} - 2\sqrt{h}\, h^{ij} V_i^+ V_j^-\right] \tag{A.1}$$

where

$$V_i^a \equiv \partial_i X^m e_m^a, \quad V_i^\pm \equiv \partial_i X^m e_m^\pm, \tag{A.2}$$

with

$$V_i^+ = \frac{1}{2}(V_i^0 + V_i^3) \tag{A.3}$$

Putting the quadratic fermionic terms and the quartic fermionic terms together one gets

$$\begin{aligned}
&S^{(2)} + S^{(4)} \\
&\quad = \frac{1}{\pi}\int d^2\sigma \; -2i\sqrt{h}\, h^{ij} V_i^+ (\theta^1 \partial_j \theta^1 + \theta^2 \partial_j \theta^2) \\
&\qquad -\frac{i}{2}\sqrt{h}\, h^{ij} \partial_j X^{\tilde{M}} w_{\tilde{M}\tilde{C}\tilde{D}} V_i^+ (\theta^1 \sigma^{\tilde{C}\tilde{D}} \theta^1 + \theta^2 \sigma^{\tilde{C}\tilde{D}} \theta^2) \\
&\qquad -2i\, \varepsilon^{ij} V_i^+ (\theta^2 \partial_j \theta^2 - \theta^1 \partial_j \theta^1) \\
&\qquad -\frac{i}{2}\, \varepsilon^{ij} \partial_j X^{\tilde{M}} w_{\tilde{M}\tilde{C}\tilde{D}} V_i^+ (\theta^2 \sigma^{\tilde{C}\tilde{D}} \theta^2 - \theta^1 \sigma^{\tilde{C}\tilde{D}} \theta^1) \\
&\qquad +\frac{i}{2} \varepsilon^{ij} V_i^+ V_j^{\tilde{D}} G^{-+\tilde{C}}{}_{\tilde{A}\tilde{B}} (\theta^1 \sigma^{\tilde{A}\tilde{B}} \theta^2 + \theta^2 \sigma^{\tilde{A}\tilde{B}} \theta^1)\, \eta_{\tilde{C}\tilde{D}} \\
&\qquad +\sqrt{-h}\, h^{ij} V_i^+ V_j^+ \Big\{ \frac{23}{576} (\theta^1\theta^2)^2\, G^{-+\tilde{A}\tilde{B}\tilde{C}} G^{-+}{}_{\tilde{A}\tilde{B}\tilde{C}} \\
&\qquad\qquad -\frac{1}{4608} (\theta^t \sigma^{\tilde{A}\tilde{B}} \theta^t)\, (\theta^s \sigma_{\tilde{A}\tilde{B}} \theta^s) G^{-+\tilde{C}\tilde{D}\tilde{E}} G^{-+}{}_{\tilde{C}\tilde{D}\tilde{E}} \\
&\qquad\qquad +0 \\
&\qquad\qquad -\frac{1}{768} (\theta^t \sigma^{\tilde{A}\tilde{B}} \theta^t)(\theta^s \sigma^{\tilde{C}\tilde{D}} \theta^s) \Big[G^{-+\tilde{E}}{}_{\tilde{A}\tilde{B}} G^{-+}{}_{\tilde{E}\tilde{C}\tilde{D}} - \frac{1}{24} G_{\tilde{A}\tilde{B}\tilde{E}\tilde{F}\tilde{G}} G^{\tilde{E}\tilde{F}\tilde{G}}{}_{\tilde{C}\tilde{D}} \Big] \\
&\qquad\qquad +\frac{1}{128} (\theta^t \sigma^{\tilde{A}\tilde{C}} \theta^t)(\theta^s \sigma^{\tilde{B}}{}_{\tilde{C}} \theta^s) \Big[G^{-+}{}_{\tilde{A}\tilde{D}\tilde{E}} G^{-+\tilde{D}\tilde{E}}{}_{\tilde{B}} - \frac{1}{72} G_{\tilde{A}\tilde{D}\tilde{E}\tilde{F}\tilde{G}} G^{\tilde{D}\tilde{E}\tilde{F}\tilde{G}}{}_{\tilde{B}} \Big] \\
&\qquad\qquad -\frac{1}{48} D_{\tilde{C}} G^{-+\tilde{C}}{}_{\tilde{A}\tilde{B}}\, \theta^1\theta^2\, (\theta^t \sigma^{\tilde{A}\tilde{B}} \theta^t) \\
&\qquad\qquad +\frac{5}{4} (\theta^1\theta^2)^2\, R^{-+-+} + \frac{1}{96} (\theta^t \sigma^{\tilde{A}\tilde{B}} \theta^t)(\theta^s \sigma_{\tilde{A}\tilde{B}} \theta^s) R^{-+-+}
\end{aligned}$$

JHEP08(2008)026

$$+\frac{1}{48}(\theta^t\sigma^{\tilde A\tilde C}\theta^t)\ (\theta^s\sigma^{\tilde B}{}_{\tilde C}\theta^s)\left[R^{-+}{}_{\tilde A\tilde B}-\frac{1}{2}R_{\tilde A\tilde C\tilde B\tilde D}\eta^{\tilde C\tilde D}\right]$$
$$+\frac{1}{192}(\theta^t\sigma^{\tilde A\tilde B}\theta^t)\ (\theta^s\sigma^{\tilde C\tilde D}\theta^s)\left[R_{\tilde A\tilde C\tilde B\tilde D}+\frac{1}{2}R_{\tilde A\tilde B\tilde C\tilde D}\right]\Bigg\} \qquad (A.4)$$

Due to the light-cone gauge constraint each fermionic coordinate θ^t has only eight non-zero components. Replacing the 16 by 16 gamma matrices $\sigma^{\tilde A}$ by 8 by 8 matrices $\gamma^{\tilde A}$ one gets, after

$$X^+=\frac{p^0+p^3}{2},\quad \theta\theta\to\frac{1}{2p^+}SS, \qquad (A.5)$$

$$\begin{aligned}
&S^{(2)}+S^{(4)}\\
&\quad=\frac{1}{\pi}\int d^2\sigma\ \frac{1}{2p^+}\Bigg\{-2i\sqrt{h}\ h^{ij}V_i^+(S^t\partial_jS^t)\\
&\qquad-\frac{i}{2}\sqrt{h}\ h^{ij}\partial_jX^{\tilde M}w_{\tilde M\tilde C\tilde D}V_i^+(S^t\sigma^{\tilde C\tilde D}S^t)\\
&\qquad-2i\ \varepsilon^{ij}V_i^+(S^2\partial_jS^2-S^1\partial_jS^1)\\
&\qquad-\frac{i}{2}\ \varepsilon^{ij}\partial_jX^{\tilde M}w_{\tilde M\tilde C\tilde D}V_i^+(S^2\sigma^{\tilde C\tilde D}S^2-S^1\sigma^{\tilde C\tilde D}S^1)\\
&\qquad+\frac{i}{2}\varepsilon^{ij}V_i^+V_j^{\tilde D}G^{-+\tilde C}{}_{\tilde A\tilde B}(S^2\sigma^{\tilde A\tilde B}S^1+S^1\sigma^{\tilde A\tilde B}S^2)\ \eta_{\tilde C\tilde D}\Bigg\}\\
&\quad+\frac{1}{4(p^+)^2}\sqrt{-h}\ h^{ij}V_i^+V_j^+\Bigg\{\frac{23}{576}(S^1S^2)^2\ G^{-+\tilde A\tilde B\tilde C}G^{-+}{}_{\tilde A\tilde B\tilde C}\\
&\qquad-\frac{1}{4608}(S^t\sigma^{\tilde A\tilde B}S^t)\ (S^s\sigma_{\tilde A\tilde B}S^s)G^{-+\tilde C\tilde D\tilde E}G^{-+}{}_{\tilde C\tilde D\tilde E}\\
&\qquad+0\\
&\qquad-\frac{1}{768}(S^t\sigma^{\tilde A\tilde B}S^t)(S^s\sigma^{\tilde C\tilde D}S^s)\left[G^{-+\tilde E}{}_{\tilde A\tilde B}G^{-+}{}_{\tilde E\tilde C\tilde D}-\frac{1}{24}G_{\tilde A\tilde B\tilde E\tilde F\tilde G}G^{\tilde E\tilde F\tilde G}{}_{\tilde C\tilde D}\right]\\
&\qquad+\frac{1}{128}(S^t\sigma^{\tilde A\tilde C}S^t)(S^s\sigma^{\tilde B}{}_{\tilde C}S^s)\left[G^{-+}{}_{\tilde A\tilde D\tilde E}G^{-+\tilde D\tilde E}{}_{\tilde B}-\frac{1}{72}G_{\tilde A\tilde D\tilde E\tilde F\tilde G}G^{\tilde D\tilde E\tilde F\tilde G}{}_{\tilde B}\right]\\
&\qquad-\frac{1}{48}D_{\tilde C}G^{-+\tilde C}{}_{\tilde A\tilde B}\ S^1S^2\ (S^t\sigma^{\tilde A\tilde B}S^t)\\
&\qquad+\frac{5}{4}(S^1S^2)^2\ R^{-+-+}+\frac{1}{96}(S^t\sigma^{\tilde A\tilde B}S^t)(S^s\sigma_{\tilde A\tilde B}S^s)R^{-+-+}\\
&\qquad+\frac{1}{48}(S^t\sigma^{\tilde A\tilde C}S^t)\ (S^s\sigma^{\tilde B}{}_{\tilde C}S^s)\left[R^{-+}{}_{\tilde A\tilde B}-\frac{1}{2}R_{\tilde A\tilde C\tilde B\tilde D}\eta^{\tilde C\tilde D}\right]\\
&\qquad+\frac{1}{192}(S^t\sigma^{\tilde A\tilde B}S^t)\ (S^s\sigma^{\tilde C\tilde D}S^s)\left[R_{\tilde A\tilde C\tilde B\tilde D}+\frac{1}{2}R_{\tilde A\tilde B\tilde C\tilde D}\right]\Bigg\}
\end{aligned} \qquad (A.6)$$

The IIB super-gravity solution for the D3 brane configuration is given by

$$\begin{aligned}
ds^2&=H^{-1/2}(dx^\mu)^2+H^{1/2}(dx^m)^2\\
G_{\bar0\bar1\bar2\bar3c}&=-\frac{X^c}{r}H^{-5/4}H'\\
G_{abcde}&=\frac{H^{-5/4}H'}{r}\varepsilon_{abcdef}X^f
\end{aligned} \qquad (A.7)$$

JHEP08(2008)026

The bar indicates that the indices are flat. Substituting the explicit forms of the connection, the five form and the Riemann tensor into the total action one gets

$$
\begin{aligned}
&S^{(0)} + S^{(2)} + S^{(4)} \\
&\quad = \frac{1}{\pi}\int d^2\sigma - \left[\frac{1}{2}\sqrt{h}\, h^{ij} V_i^{\tilde{A}} V_{\tilde{A}j} - 2\sqrt{h}\, h^{ij} V_i^{+} V_j^{-}\right] \\
&\quad + \frac{1}{2p^+}\Big\{ -2i\sqrt{h}\, h^{ij} V_i^{+}(S^t \partial_j S^t) - 2i\varepsilon^{ij} V_i^{+}(S^2\partial_j S^2 - S^1 \partial_j S^1) \\
&\qquad + \frac{i}{4}\sqrt{h} h^{ij} V_i^{+} H^{-3/2}\frac{H'}{r}\partial_j X^u X^m S^t \gamma^{um} S^t \\
&\qquad - \frac{i}{4}\sqrt{h} h^{ij} V_i^{+} H^{-1}\frac{H'}{r}\partial_j X^m X^n S^t \gamma^{mn} S^t \\
&\qquad + \frac{i}{4} V_i^{+}\varepsilon^{ij} H^{-3/2}\frac{H'}{r}\partial_j X^u X^m (S^2\gamma^{um}S^2 - S^1\gamma^{um}S^1) \\
&\qquad - \frac{i}{4} V_i^{+}\varepsilon^{ij} H^{-1}\frac{H'}{r}\partial_j X^m X^n (S^2\gamma^{mn}S^2 - S^1\gamma^{mn}S^1) \\
&\qquad + i\varepsilon^{ij} V_i^{+} H^{-5/4}\frac{H'}{r} X^b (V_j^b S^2\gamma^{12}S^1 + V_j^1 S^2 \gamma^{2b} S^1 - V_j^2 S^2 \gamma^{1b} S^1)\Big\} \\
&\quad + \frac{1}{4(p^+)^2}\sqrt{h} h^{ij} V_i^{+} V_j^{+}\ \Big\{\frac{23}{24}(S^1 S^2)^2 H^{-5/2}(H')^2 \\
&\qquad - \frac{1}{192}(S^t\gamma^{\tilde{A}\tilde{B}}S^t)(S^{t'}\gamma_{\tilde{A}\tilde{B}}S^{t'}) H^{-5/2}(H')^2 \\
&\qquad - \frac{1}{768} H^{-5/2}(H')^2\Big[16(S^t\gamma^{12}S^t)(S^{t'}\gamma^{12}S^{t'}) + 16(S^t\gamma^{au}S^t)(S^{t'}\gamma^{cu}S^{t'})\frac{X^a X^c}{r^2} \\
&\qquad\qquad - \frac{1}{24}(S^t\gamma^{ab}S^t)(S^{t'}\gamma^{cd}S^{t'})\frac{X^h X^{h'}}{r^2}\varepsilon_{abefgh}\varepsilon_{cdefgh'}\Big] \\
&\qquad + \frac{1}{16} H^{-5/2}(H')^2\Big[(S^t\gamma^{a\tilde{C}}S^t)(S^{t'}\gamma^{b}{}_{\tilde{C}}S^{t'})\frac{X^a X^b}{r^2} + (S^t\gamma^{au}S^t)(S^{t'}\gamma^{au}S^{t'}) \\
&\qquad\qquad - \frac{1}{576}(S^t\gamma^{a\tilde{C}}S^t)(S^{t'}\gamma^{b}{}_{\tilde{C}}S^{t'})\frac{X^h X^{h'}}{r^2}\varepsilon_{adefgh}\varepsilon_{bdefgh'}\Big] \\
&\qquad + \frac{1}{24} H^{-5/2}(H')^2\ S^1 S^2 (S^t\gamma^{12}S^t) + \frac{5}{16} H^{-5/2}(H')^2\ S^1 S^2 \\
&\qquad + \frac{1}{384} H^{-5/2}(H')^2\ (S^t\gamma^{\tilde{A}\tilde{B}}S^t)(S^{t'}\gamma_{\tilde{A}\tilde{B}}S^{t'}) \\
&\qquad + \frac{3}{1536} H^{-5/2}(H')^2 (S^t\gamma^{\alpha\tilde{C}}S^t)(S^{t'}\gamma_{\alpha\tilde{C}}S^{t'}) \\
&\qquad - \frac{1}{96}(4g_2 X^a X^b + 4g_3\delta_{ab})(S^t\gamma^{a\tilde{C}}S^t)(S^{t'}\gamma^{b}{}_{\tilde{C}}S^{t'}) \\
&\qquad - \frac{1}{384}(4g_2 r^2 + 24 g_3)\ (S^t\gamma^{\alpha\tilde{C}}S^t)(S^{t'}\gamma_{\alpha\tilde{C}}S^{t'}) \\
&\qquad - \frac{1}{96}(4h_1 X^a X^b + h_1 r^2\delta_{ab} + 5h_2\delta_{ab})(S^t\gamma^{a\tilde{C}}S^t)(S^{t'}\gamma^{b}{}_{\tilde{C}}S^{t'}) \\
&\qquad + \frac{1}{96} g_1 (S^t\gamma^{\alpha\beta}S^t)(S^{t'}\gamma_{\alpha\beta}S^{t'}) + \frac{1}{48}(g_2 X^a X^b + g_3\delta_{ab})(S^t\gamma^{\alpha a}S^t)(S^{t'}\gamma_{\alpha}{}^{b}S^{t'}) \\
&\qquad + \frac{1}{96}\Big[2h_1 X^a X^b (S^t\gamma^{ae}S^t)(S^{t'}\gamma_{be}S^{t'}) + h_2 (S^t\gamma^{ab}S^t)(S^{t'}\gamma_{ab}S^{t'})\Big]\Big\}
\end{aligned}
\tag{A.8}
$$

JHEP08(2008)026

When we compute the amplitude below we will use the dimensional regularization. Considering that the scattering states contain only the S^1 coordinate but not S^2 we can drop the terms in (A.8) that have an S^2 factor. Defining

$$S \equiv S^1 \tag{A.9}$$

and setting $V_j^- = 0$, one gets after some algebra

$$\begin{aligned}
&S^{(0)} + S^{(2)} + S^{(4)} \\
&\quad = \frac{1}{\pi} \int d^2\sigma \; -\frac{1}{2}\sqrt{h}\, h^{ij} \left(\partial_i X^u \partial_j X^v \eta_{uv} H^{-1/2} + \partial_i X^m \partial_j X^n \eta_{mn} H^{1/2}\right) \\
&\quad + \frac{1}{2p^+}\Big\{ -2i(\sqrt{h}\, h^{ij} - \varepsilon^{ij})\partial_i X^+ H^{-1/4}(S\partial_j S) \\
&\qquad + \frac{i}{4}(\sqrt{h}h^{ij} - \varepsilon^{ij})\partial_i X^+ H^{-7/4}\frac{H'}{r}\partial_j X^u X^m \; (S\gamma^{um}S) \\
&\qquad - \frac{i}{4}(\sqrt{h}h^{ij} - \varepsilon^{ij})\partial_i X^+ H^{-5/4}\frac{H'}{r}\partial_j X^m X^n \; (S\gamma^{mn}S)\Big\} \\
&\quad + \frac{1}{4(p^+)^2}\sqrt{h}h^{ij}\partial_i X^+ \partial_j X^+ \; H^{-1/2}\Big\{ -\frac{17}{1536}\kappa_1 (S\gamma^{uv}S)(S\gamma^{uv}S) \\
&\qquad + \left[\frac{43}{768}\kappa_1 + \frac{1}{192}\kappa_2\right](S\gamma^{au}S)(S\gamma^{au}S) \\
&\qquad - \left[\frac{1}{192}\kappa_2 + \frac{1}{128}\kappa_1\right](S\gamma^{ab}S)(S\gamma^{ab}S) \\
&\qquad + X^a X^b \frac{1}{r^2}\left[\frac{31}{768}\kappa_1 - \frac{1}{32}\kappa_2\right](S\gamma^{au}S)(S\gamma^{bu}S) \\
&\qquad + X^a X^b \frac{1}{r^2}\left[+\frac{1}{32}\kappa_2 + \frac{29}{384}\kappa_1\right](S\gamma^{ac}S)(S\gamma^{bc}S)\Big\}
\end{aligned} \tag{A.10}$$

where

$$\kappa_1 = H^{-5/2}(H')^2, \qquad \kappa_2 = H^{-3/2}H'\frac{1}{r} \tag{A.11}$$

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JHEP08(2008)026